Prediction Based Solution to the Consensus Problem for Mobile Robots with Communication Time Delay

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Abstract—This work presents the results of the experimental proofs for the consensus protocol previously proposed by the authors. The protocol compensates the communication time delay of a multiagent system of first order dynamics agents. The communication time delay is modeled as input delay. The delay of a multiagent system of first order dynamics agents is compensated by means of a state predictor in [14]. The input time delay of a linear MIMO (Multiple Input Multiple Output) agents is compensated by means of a state predictor in [14].

Index Terms—Consensus problem, mobile robots, time delay, prediction-observation.

I. INTRODUCTION

Multiagent systems have numerous applications such as satellite clusters, search operations, formation of unmanned vehicles distributed sensors and others [1]–[3]. Consensus problem is a basic problem of distributed systems. There exist three important factors that affect the achievement of consensus, these are communication topology, time delays and the model of the agents [4]–[6].

Communication among agents has been widely studied and the importance of the Laplacian matrix has been stated [1], [3], [7]. The consensus problem in presence of communication delay has been widely studied and numerous solutions have been presented. In particular, the presence of communication delays may provoke the system to become unstable. Such solutions have considered a wide range of conditions as fixed and variable time delay, fixed and variable topologies, and multiple time delays [4], [8], [9], [10], [11].

Prediction based schemes have been proposed for systems without time delays to estimate the current state for systems where only a partial measurement of the state is possible. The consensus problem of single integrator agents with input and output time delay is studied in [12], while the formation problem of double integrator agents is solved by means of a consensus based scheme in [13]. The input time delay of a heterogeneous system formed by single and double integrator agents is compensated by means of a state predictor in [14]. A set of chained observers is proposed in [15] to control and stabilize linear MIMO (Multiple Input Multiple Output) systems with time delay.

This work deals with the consensus problem of a set of mobile agents with communication delay. The main objective of the note is to present the experimental performance of the prediction-based consensus scheme presented in [16]. The agents of the system are a set of (2, 0) mobile robots. Since the consensus protocol is designed for first order agents, the kinematic model of a point outside of the axis of the wheels is considered. The communication among agents is assumed to be either undirected or directed with a spanning tree.

The paper is organized as follows. Section II presents the kinematic model of the agents. The problem statement is given in Section III. The predictor-observer and control schemes are shown in Sections IV and V respectively. The experimental results can be found in Section VI. The work ends with some conclusions presented in Section VII.

II. KINEMATIC MODEL OF THE UNICYCLE MOBILE ROBOT

The representation of the unicycle mobile robots considered in this work is given in Fig. 1. The robots consist of two fixed wheels on the same axis and with independent motors. The kinematic model of each robot is given by,

\[
\begin{bmatrix}
\dot{x}_i(t) \\
\dot{y}_i(t) \\
\dot{\theta}_i(t)
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_i & 0 & v_i \\
\sin \theta_i & 0 & 0 \\
0 & 1 & \omega_i
\end{bmatrix}
\]

(1)

where \(x_i\) and \(y_i\) represent the position of the center of the axis of the wheels, and \(\theta_i\) the orientation of each robot with respect to the fixed plane \(X - Y\), \(v_i\) is the linear velocity and \(\omega_i\) represents the angular velocity of the robot. Since system (1) is nonlinear, a point \(\alpha_i\) outside of the wheels axis with linear kinematics is considered.

The position of the point \(\alpha_i\) is given by,

\[
\alpha_i =
\begin{bmatrix}
p_i \\
q_i
\end{bmatrix} =
\begin{bmatrix}
x_i + l \cos \theta_i \\
y_i + l \sin \theta_i
\end{bmatrix}
\]

(2)

where \(l\) is the distance from the center of the wheel axis and the point \(\alpha_i\). The kinematics of the point \(\alpha_i\) are given by,

\[
\dot{\alpha}_i(t) = A_i(\theta_i)
\]

(3)

with the decoupling matrix \(A_i(\theta_i)\) given by,

\[
A_i(\theta_i) =
\begin{bmatrix}
\cos \theta_i & -l \sin \theta_i \\
\sin \theta_i & l \cos \theta_i
\end{bmatrix}
\]

(4)
Defining a new control signal, $u_i = [u_{i1}, u_{i2}]^T$, the dynamics of each agent are given by

$$\dot{x}_i(t) = a_ix_i(t) + b_iu_i(t - \tau_i)$$

(9)

for $i = 1, 2, \ldots, n$, $x_i \in \mathbb{R}$ represents the state of the $i$-th agent, $u_i \in \mathbb{R}$ the input signal, $a_i$ and $b_i$ are constant parameters, and $\tau_i > 0$ the known and constant input time delay.

**Consensus problem:** A system of $n$ agents is said to achieve consensus if for any initial conditions $x_i(0); x_i(t) \rightarrow x_r(t)$ as $t \rightarrow \infty$ for all $i, r = 1, \ldots, n$ [2]. Where $x_i(t)$ and $x_r(t)$ represent the states of any two agents of the system.

**III. PROBLEM STATEMENT**

The objective of a consensus protocol is to impose the same dynamics to each agent. This work considers multiagent systems with first order dynamics and input time delay and fixed communication topology. The graph that represents the communication topology can be either undirected or directed with a spanning tree, which results in Laplacian matrices with one eigenvalue equal to zero [17].

Let be the agents dynamics given by,

$$\dot{x}_i(t) = a_ix_i(t) + b_iu_i(t - \tau_i)$$

(9)

for $i = 1, 2, \ldots, n$, $x_i \in \mathbb{R}$ represents the state of the $i$-th agent, $u_i \in \mathbb{R}$ the input signal, $a_i$ and $b_i$ are constant parameters, and $\tau_i > 0$ the known and constant input time delay.

**Lemma 1:** Consider a system of the form (9), if the estimator errors converge asymptotically to the real future states $x(t + j\bar{\tau}_i)$.

This is, $\hat{w}_{ij}(t) \rightarrow w_{ij}(t)$ when $t \rightarrow \infty$.

**Proof:** The dynamics of the prediction errors (15) are given by

$$\dot{\hat{e}}_{wij}(t) = a_i\hat{e}_{wij}(t) - \lambda_i\hat{e}_{wij}(t - \bar{\tau}_i)$$

(16)

for $j, k = 1, \ldots, m$. Equation (16) shows that the convergence to zero of errors (15) guarantees the convergence of the advanced estimated states to the real future values. The following lemma states the convergence of the predicted states to the future states of the system.

**Lemma 1:** Consider a system of the form (9), if the predictor-observer scheme (14) satisfies (10), then there exists $\lambda_i$ for $i = 1, \ldots, n$ such that the estimated future states $\hat{w}_{ij}(t)$ converge asymptotically to the real future states $x(t + j\bar{\tau}_i)$.

This is, $\hat{w}_{ij}(t) \rightarrow w_{ij}(t)$ when $t \rightarrow \infty$.

**Proof:** The dynamics of the prediction errors (15) are given by

$$\dot{\hat{e}}_{wij}(t) = a_i\hat{e}_{wij}(t) - \lambda_i\hat{e}_{wij}(t - \bar{\tau}_i)$$

(16)
this is,
\[ \dot{e}_{wi}(t) = A_0 e_{wi}(t) + A_1 e_{wi}(t - \bar{\tau}_i) \]  
(18)
with the matrix \( A_0 \in \mathbb{R}^{m \times m} \) given by,
\[
A_0 = \begin{bmatrix}
    a_{i} & 0 & \cdots & 0 \\
    \lambda_i & a_{i} & \cdots & 0 \\
    0 & \lambda_i & \ddots & \vdots \\
    \vdots & 0 & \ddots & \lambda_i \ni a_{i} \\
    0 & \cdots & 0 & \lambda_i \ni a_{i}
  \end{bmatrix}
\]
the matrix \( A_1 = \lambda I_{m} \in Re^{m \times m} \), and the vector \( e_{wi} = [e_{wi1}, e_{wi2}, \ldots, e_{wim}]^T \). The characteristic equation of (18) is given by \( p(s) = \det(sI - A_0 - A_1 e^{-s\bar{\tau}_i}) = 0 \), is given by,
\[
p(s) = \prod_{j=1}^{m} \left( s - a_i + \lambda_i e^{-s\bar{\tau}_i} \right) = 0. \quad (19)
\]

The analysis of the characteristic equation (19) allows to establish that system (18) is asymptotically stable when \( \tau_i \leq \tau^* \) [18] with \( \tau^* = \frac{\arccos \left( \frac{\tau_i}{\sqrt{\lambda^2 - a_i^2}} \right)}{\sqrt{\lambda^2 - a_i^2}} \).

Remark 1: Note that the predictor-observer (14) is designed in terms of the delayed prediction error signals \( e_{wi}(t - \tau_i) \) to guarantee the causality of the scheme.

V. CONSENSUS PROTOCOL

The consensus protocol based on predicted future states of the robots is briefly presented in this section. For a set of \( n \) agents of the form (9) with the same input delay, the consensus protocol is given by,
\[
u_i(t) = \frac{1}{b} \left[ a_{i} \dot{w}_{im} + \sum_{r=1}^{n} a_{ir}(\dot{w}_{ir}(t) - \dot{w}_{rm}(t)) \right] \quad (20)
\]
for \( i, r = 1, 2, \ldots, n \) and \( a_{ir} \) the elements of the adjacency matrix. For a system with \( n \) agents with the same kinematics and communication delay given by,
\[ \dot{x}(t) = A_x x(t) + Bu(t - \tau) \]  
(21)
with \( x = [x_1, \ldots, x_n]^T, \ u = [u_1, \ldots, u_n]^T, \ \tau = \tau_1 = \ldots = \tau_n, \ A_x = \text{diag}[a_1, \ldots, a_n], \) and \( B = \text{diag}[b_1, \ldots, b_n]. \) The vectorial form of the consensus control (20) for system (21) is given by,
\[ u(t) = B^{-1} (A_x \dot{w}_m(t - \tau) - \bar{\mathcal{L}} \dot{w}_m(t - \tau)) \]  
(22)
where \( \bar{\mathcal{L}} \) is the Laplacian matrix associated to the communication topology of the system. The closed loop system (21)-(22) results,
\[ \dot{x}(t) = A_x (x(t) - \dot{w}_m(t - \tau)) - \bar{\mathcal{L}} \dot{w}_m(t - \tau). \quad (23) \]
From (15), \( \dot{w}_m(t - \tau) \) can be rewritten as,
\[
\dot{w}_m(t - \tau) = x(t) - \sum_{k=1}^{m} e_{wk}(t - k\tau)
\]
with \( e_{wk} = [e_{wk1}, \ldots, e_{wkn}]^T \) that allows rewriting (23) as,
\[ \dot{x}(t) = -\mathcal{L} x(t) + (A_x + \mathcal{L}) \sum_{k=1}^{m} e_{wk}(t - k\tau) \]  
(24)
for \( k = 1, \ldots, m. \)

Lemma 2: Consider a set of \( n \) agents with the kinematic model given by (3) and future estimated state obtained by the predictor-observer scheme (14) and assume that the communication topology of the agents is undirected or directed with a spanning tree. Then, for any \( \tau > 0 \), there exists \( m > 0 \) such that protocol (22) guarantees consensus of the agents.

Proof: Consider a non singular matrix \( P = [v_1 \ldots v_n] \in \mathbb{R}^{n \times n} \) where the columns \( v_i \) correspond to the eigenvectors of the Laplacian matrix \( \bar{\mathcal{L}} \). Then, considering the variable change \( x = P \chi, \ \chi = P^{-1} x \), the closed-loop system (24) can be rewritten as,
\[ \dot{\chi}(t) = J \chi(t) + P^{-1} (A_x + \mathcal{L}) \sum_{k=1}^{m} e_{wk}(t - k\tau) \]  
(25)
where \( J = P^{-1} \bar{\mathcal{L}} P \) is the canonical Jordan form of the Laplacian matrix \( \bar{\mathcal{L}} \). Solution to system (25) is given by,
\[ \chi(t) = e^{-J t} \chi(0) + \sum_{k=1}^{m} \int_{0}^{t} e^{-J(t - \theta)} P^{-1} \mathcal{L} e_{wk}(\theta - k\tau) d\theta \]
(26)
Since the communication topology is either undirected or directed with a spanning tree, the first eigenvalue of \( \mathcal{L} \) is \( \nu_1 = 1_n \), with \( 1_n = [1, \ldots, 1]^T \in \mathbb{R}^n \). Then,
\[ J = P^{-1} \mathcal{L} P = \begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix} \]  
(27)
Considering agents with the same dynamics, this is \( A_x = \text{diag}(a) = a I_n \), noticing that \( JP^{-1} = P^{-1} \mathcal{L} \) and after some simplifications (26) results,
\[ \dot{\chi}_1(t) = \chi(0) + a \sum_{k=1}^{m} \int_{0}^{t} P_{ci}^{-1} e_{wk}(\theta - k\tau) d\theta \]
\[ \dot{\chi}_2(t) = e^{-J t} \chi(0) + \sum_{k=1}^{m} \int_{0}^{t} e^{J \theta} P^{-1} \mathcal{L} e_{wk}(\theta - k\tau) d\theta + \sum_{k=1}^{m} \int_{0}^{t} e^{J \theta} P^{-1} A_x e_{wk}(\theta - k\tau) d\theta \]
(28)
with \( \dot{\chi}_1 = \chi_1 \) and \( \dot{\chi}_2 = [\chi_2, \ldots, \chi_n]^T \), and where \( P_{ci} \in \mathbb{R}^{n-1 \times n} \) is the matrix formed by the rows of \( P^{-1} \) except for the first one, \( P^{-1} = [P_{c1}^{-1} P_{ci}^{-1}]^T \). From (28) it is easy to see that,
\[ \lim_{t \to \infty} \dot{\chi}_2(t) = 0. \quad (29) \]

Then, considering \( x(t) = P \chi(t) \) and equation (29), it can be concluded that,
\[ \lim_{t \to \infty} x(t) = P \begin{bmatrix} 1 & 0 \\ 0_{(n-1) \times (n-1)} & I \end{bmatrix} \chi(0) + P \begin{bmatrix} a P_{c1}^{-1} \\ 0_{(n-1) \times n} \end{bmatrix} k_x \]
with \( k_s = \lim_{t \to \infty} \sum_{k=1}^{m} \int_{0}^{t} e_{wk}(\theta - k\tau) d\theta \). Recalling that the first column of \( P \) is the vector \( 1_n \),

\[
P \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0_{(n-1) \times (n-1)} & 0 \end{bmatrix} \chi(0) = \begin{bmatrix} P_{11}^{-1} x(0) \\ \vdots \\ P_{1m}^{-1} x(0) \end{bmatrix}
\]

that shows that the states of all the agents \( x_i(t) \), for \( i = 1, \ldots, n \) converge to the same value,

\[
x_i(\infty) = P_{1i}^{-1} x(0) + a P_{1i}^{-1} \lim_{t \to \infty} \sum_{k=1}^{m} \int_{0}^{t} e_{wk}(\theta - k\tau) d\theta
\]

for \( k = 1, \ldots, m \).

VI. EXPERIMENTAL EVALUATION

The results of the experimental evaluations are presented in this section. The results show that the prediction based control scheme allows achieving consensus in the presence of large time delays.

A. Experimental platform

The performance of the predictor-based consensus protocol (22) was evaluated on an experimental platform formed by three unicycle robots, an image processing system and a computer with ROS (Robot Operating System).

The agents are three TurtleBot3 mobile robots from ROBOTIS. The TrutleBot3 is a ROS standard platform robot. The model of two of the robots is called Burger and the third is called Waffle. The localization system is an Optitrack motion capture system that consists of an array of 12 Flex-13 cameras from Natural Point Inc., and Motive, a software that processes the information sent by the cameras and retrieves the position and orientation of the robots. The information is sent to a computer equipped with ROS through a VRPN (Virtual-Reality Peripheral Network). With the received information, the computer estimates the future state values and the corresponding control signals and sends them to each robot. The time delay is set artificially. The communication topology of the system is depicted in Fig. 2.

![Fig. 2. Communication Topology.](image)

B. Adaptation of the predictor-observer and the consensus control schemes

The kinematic model of the robots (8) is a first order system that can be written in the form (9), with \( a = 0 \) and \( b = 1 \). Thus, the predictor-observer scheme (14) can be rewritten as,

\[
\dot{\hat{w}}_{ij}(t) = u_i(t - (m - j)\tau_i) + \lambda_{ij} e_{wij}(t - \tau)
\]

for \( j = 1, \ldots, m \). The condition for the maximum delay compensated by each subpredictor results,

\[
\tau^* = \frac{\pi}{2\lambda_{ij}}
\]

The consensus scheme 20, is also rewritten as,

\[
u_i(t) = \sum_{r=1}^{n} a_{ir}(\hat{w}_{irm}(t) - \hat{w}_{rm}(t))
\]

for \( i, r = 1, \ldots, n \). The consensus value then is given by,

\[
x_i(\infty) = P_{1i}^{-1} x(0)
\]

that is the average of the initial conditions of the agents. To avoid collision of the robots at the consensus point, an equivalent triangle formed by the point outside the axis of the robot wheels \( \alpha_i \) was set as consensus criteria. To do so, the following modification was made to the consensus protocol (33),

\[
u_i(t) = \sum_{r=1}^{n} a_{ir}(\hat{w}_{irm}(t) - \hat{w}_{rm}(t) - c_{ir})
\]

where \( c_{ir} \) denotes the relative position between agents \( i \) and \( r \).

C. Experimental results

Two results are presented here, for the sake of simplicity in both cases the constant gains of the sub-predictors \( \lambda_{ij} \) are set to be the same for all the agents and the number of partitions of the predictor-observer scheme is \( m = 4 \). Arbitrary initial conditions of the robots and the predictors were used.

For an input time delay \( \tau_i = 0.8s \), the gain of the sub-predictors used was set to \( \lambda_{ij} = \pi/2 \). Fig. 3 depicts the evolution in time of the control signals sent to the mobile robots. The evolution of the prediction errors are presented in Fig. 4, the linear and angular velocities of the robots are presented in Fig. 5 and the trajectories followed by the robots are presented in Fig. 6.

In a considerably more challenging situation, a time delay \( \tau_i = 2.4s \) was considered. In this case, the sub-predictors gain is \( \lambda_{ij} = \pi/6 \). The calculated control signals are presented in Fig. 7. Fig. 8 presents the evolution in time of the prediction errors, the evolution in time of the velocities of the agents can be found in Fig. 9 and Fig. 10 presents the plane motion of the robots.

As it can be seen from the results, the proposed prediction based consensus protocol is capable of compensating large time delays. It can also be observed that a smaller \( \lambda_{ij} \) allows a faster convergence of the prediction errors and therefore a faster achievement of consensus, while a bigger value allows compensating a larger time delay but results in a slower convergence of the predicted advanced states.
Fig. 3. Time evolution of the control signals. $\tau = 0.8s$

Fig. 4. Time evolution of the prediction error signals. $\tau = 0.8s$

Fig. 5. Time evolution of the agents velocities. $\tau = 0.8s$

Fig. 6. $X - Y$ plane motion of the robots. $\tau = 0.8s$

Fig. 7. Time evolution of the control signals. $\tau = 2.4s$

Fig. 8. Time evolution of the prediction error signals. $\tau = 2.4s$
VII. Conclusions

Experimental results that validate the performance of the consensus protocol based on predicted states are presented in this work. It is shown that the partitioned nature of the predictor-observer that estimates the future states of the agents, allows compensating larger communication delays. The proofs allow to conclude that for a first order system, a small number of partitions of the predictor-observer is enough to compensate large time delays. Larger gains of the sub-predictors can achieve a faster convergence of the estimated values to the real future states, and smaller gain values result in a slower convergence of the predicted states but allow to compensate larger time delays.

REFERENCES